LEARNING TO SEE ANALOGIES:  
A CONNECTIONIST EXPLORATION

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For Laura and Thaddeus.
Abstract

This dissertation explores the integration of learning and analogy-making through the development of a computer program, called Analogator, that learns to make analogies by example. By “seeing” many different analogy problems, along with possible solutions, Analogator gradually develops an ability to make new analogies. That is, it learns to make analogies by analogy. This approach stands in contrast to most existing research on analogy-making, in which typically the a priori existence of analogical mechanisms within a model is assumed.

The present research extends standard connectionist methodologies by developing a specialized associative training procedure for a recurrent network architecture. The network is trained to divide input scenes (or situations) into appropriate figure and
ground components. Seeing one scene in terms of a particular figure and ground provides the context for seeing another in an analogous fashion. After training, the model is able to make new analogies between novel situations.

Analogator has much in common with lower-level perceptual models of categorization and recognition; it thus serves as a unifying framework encompassing both high-level analogical learning and low-level perception. This approach is compared and contrasted with other computational models of analogy-making. The model’s training and generalization performance is examined, and limitations are discussed.
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